Noise Robustness in Multi-Level Crossing Interval based Method of Spectral Estimation

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Abstract

A new method of sinusoid estimation in noise using multi-level crossing interval information is presented. Using the statistics of level crossing intervals a sinusoid detection criterion is formulated and an expression for threshold SNR is derived for any single level. The analysis is extended to a pooled set of multi-level crossing intervals resulting in a better SNR threshold of detecting a sinusoid. It is demonstrated that the multi-level crossing information is more robust than single level crossing information and the robustness improves with increasing contribution from higher levels.

Introduction

Robust spectral estimation is required in several areas of signal processing like processing of noisy speech data, biomedical signal processing etc. Currently popular SVD based methods of spectral estimation utilize prior information regarding the signal to obtain robust estimates. However, experiments have demonstrated that the spectral analysis properties of the human auditory system are far superior to these methods in the presence of noise [1]. With increased understanding of the signal processing in the auditory system, there is enhanced interest in applying some of the ideas of auditory models to spectral estimation.

The mechanical to neural transduction in the inner ear of the auditory system comprises of detectors which are sensitive to the magnitude of the basilar membrane displacement. Thus, the neural input to the central auditory system comprises of the level crossing information of the bandpass signals generated by the cochlea.

In speech processing, the zero level crossing (ZLC) information has been used effectively for robust parameter estimation [3]. The processing of multi-level crossing information of the auditory system can be viewd as a generalization of the ZLC methods. For short data records, the performance of the ZLC based methods would be limited because of under utilization of signal information. The higher level crossing information would additionally provide the amplitude information of the signal which should be effectively utilized.

This paper provides an analytical derivation of the threshold for sinusoid frequency estimation using any single level crossing (SLC) interval information. Further, by pooling the data from various levels an MLC based detection criterion is formulated and is shown to be advantageous than SLC interval information. The performance of the proposed spectral estimator for the case of a single sinusoid in noise is then compared with that of the maximum likelihood (ML) spectral estimator.

Analysis of SLCs

Consider a finite duration signal consisting of M sinusoids in white Gaussian noise.

$$x(t) = \sum_{m=1}^{M} A_m \cos(\omega_m t + \phi_m) + gv(t). \quad (1)$$

The problem is to estimate the frequencies of the sinusoids using SLC intervals of the signal without any apriori information. The method consists of utilizing the contrast between the statistics of the LC intervals of the sinsoid plus noise and only noise to localize the signal components. The statistics of the LC intervals of a multi-sinusoid signal is complicated. Hence the signal is first separated into components using a bank of bandpass filters such that either only noise or a sinusoid plus noise is obtained at the output of each filter, as given below, where k is the filter index.

$$x_s^k(t) = A_m \cos(\omega_m t + \phi_m) + gv_b(t), \quad (2)$$
$$x_v^k(t) = gv_b(t). \quad (3)$$

The filters whose output contain a sinusoid are determined and the frequency of the detected sinusoid is estimated using the dominant frequency principle [4].

Statistics of LC intervals

The successive level crossings $t_n, n = 1, 2, ...$ of a signal at any amplitude level A_l are defined as the time locations at which the signal $x(t_n) = A_l$. The LC intervals $\tau_n, n = 1, 2, \cdots$ are defined as the intervals between alternate LC instants: $\tau_n = t_{n+2} - t_n$. Because of the noise in the signal the LC interval denotes a random varible τ whose pdf is given by $p(\tau)$. Theoretical analysis of the ZLC intervals ($A_\ell = 0$) of a random function is acknowledged in literature to be complicated. Hence, the statistics of the LC intervals for any specific level and a given power spectral density is not easily obtained. However, using a perturbation analysis and computer simulations, the requisite statistical measures are obtained as below.

Sinusoid plus noise

The mean LC interval of a bandpass signal $x_s(t)$ may be expressed using the dominant frequency principle [4] as

$$\eta_s = 2\pi/\omega_m. \quad (4)$$

To calculate the variance, consider the sinusoid and additive bandpass noise as shown in Fig. 1. At an LC t_n of $x_s(t)$, we have

$$A_m cos(\omega_m t_n + \phi_n) + gv_b(t_n) = A_l, \quad (5)$$

where A_l is the amplitude of the l^{th} level. At high SNRs, the LC instants of the noise corrupted sinusoid can be viewed as a small perturbation on the LC instants of the pure sinusoid. Therfore, t_n may be expressed as $t_n = \tilde{t}_n + \delta_n$ where $\tilde{t}_n$ is the LC of the pure sinsoid and δ_n is the perturbation. Since $A_m cos(\omega_m \tilde{t}_n + \phi_n) = A_l$, we can relate the perturbation δ_n to the instantaneous noise amplitude as

$$\delta_n = \frac{gv_b(t_n)}{A_m \omega_m \sqrt{1 - r_m^2 a_l^2}}. \quad (6)$$

where $a_l = A_l/A_{max}$ and $r_m = A_{max}/A_m$. Considering two successive LC perturbations to be uncorrelated, the standard deviation of the LC intervals at a particular level is given by

$$\sigma_s(l) = \frac{2g\sqrt{b/\pi}}{A_m \omega_m \sqrt{1 - r_m^2 a_l^2}}. \quad (7)$$

Note that $\sigma_s(l)$ is dependent on ω_m also.

Bandpass noise

Analytical derivation of the $p(\tau)$ of the ZLC intervals has been reported in the literature [2]. This probability density function can be approximated as an uniform pdf, since we are interested mainly in the mean and variance measures. Thus, (see Fig. 2)

$$p_v(\tau) = \frac{\omega_c^2 - b^2}{2b\pi}, \quad \frac{\pi}{\omega_c + b} \leq \tau \leq \frac{\pi}{\omega_c - b}, \quad (8)$$

where $\omega_c, 2b$ determine the centre frequency and bandwidth of the bandpass filter. Since the successive ZLC instants are independant the mean of the intervals between alternate ZLCs would be twice the mean interval between successive ZLCs. The variance would be the same in both cases. Hence, the mean η_v and the standard deviation σ_v of the intervals between alternate ZLC instants can be obtained as

$$\eta_v = \frac{2\pi\omega_c}{\omega_c^2 - b^2}, \quad \sigma_v = \frac{\pi b}{\sqrt{3}(\omega_c^2 - b^2)}. \quad (9)$$

Similar measure as above for any general level is not available in literature. Hence, using computer simulations and a functional fitting of the data is used to obtain the requisite expressions.

For the noise signal, although it has an infinite amplitude range, substantial number of LCs occur only at levels within a certain amplitude level which is a function of the average power $E_v = g^2 2b/\pi$ of the bandpass noise. Within this finite range, it has been observed through computer simulations that η_v can be approximated to be invariant with level whereas σ_v increases with level. Simulation results suggest that σ_v for any given level $i\sqrt{E_v}$ fits the expression

$$\sigma_v(i\sqrt{E_v}) = \sigma_v(0)e^{ci}, \quad for \quad i \leq \alpha, \quad (10)$$

where c is a constant and $\alpha\sqrt{E_v}$ is the level beyond which no signifant level crossings are found. From Fig. 3, it is found that $c = 8.7$ would give a conservative fit to the simulated data.

Mapping the bandpass noise level i to level a_l of sinusoid plus noise,

$$i = \frac{a_l A_{max}}{\sqrt{E_v}} = \sqrt{\frac{\pi}{2b}}\frac{A_{max}}{g}a_l. \quad (11)$$

Thus, σ_v may be written as a function of a_l:

$$\sigma_v(l) = \frac{\pi b}{\sqrt{3}(\omega_c^2 - b^2)}e^{c\sqrt{\pi/2b}(A_{max}/g)a_l}. \quad (12)$$

Detection using SLCs

Because of the dominating sinusoid, the variance of the LC intervals of $x_s(t)$ is much smaller than that of $x_v(t)$. The detection measure for localizing the sinusoid is formulated as $\xi[k] = \eta^k/\sigma^k$, for the k^{th} filter, so as to minimize the dependence on frequency. Using the maximum aposteriori probability (MAP) criterion, the detection threshold is obtained by the condition $\eta_s(l)/\sigma_s(l) > \eta_v(l)/\sigma_v(l)$. Substituting and simplifying, we get

$$\frac{A_m\pi}{g} > \frac{\sqrt{12}\omega_c/\sqrt{\pi b}}{\sqrt{1 - r_m^2 a_l^2 e^{c\sqrt{\pi/2b}(A_{max}/g)a_l}}} \quad (13)$$

where $20log(A_m\pi/g)$ is defined as the local SNR (dB) of the sinusoid. Eqn.(13) gives the minimum local SNR required to detect a sinusoid using the level crossing information at level A_l. The SNR threshold for the ZLC case can be obtained from (13) for $a_l = 0$ as

$$\frac{A_m\pi}{g} > \frac{\sqrt{12}\omega_c}{\sqrt{\pi b}} = SNR(0). \quad (14)$$

It is clear from (13) that the sinusoid detection threshold using the SLC information of a particular level decreases with increase in the level. Although this indicates that the intervals from higher levels provide better noise robustness, it should be noted that for finite data records, the number of intervals available at higher levels is usually fewer than that available at lower levels. Thus, higher level crossing information may be less robust for short data records of $x(t)$ but more robust for sinusoid detection provided sufficiently long record of $x(t)$ is available.

Analysis of MLCs

To obtain a better spectral estimator for short data records, the LC information from different levels should be combined for optimum performance. Statistical consistancy of the interval statistics and the degree of noise robustness at a level may be used as the parameters to determine the weightage given to the LC intervals at a particular level. As a first step in this direction, the LC intervals from different levels may be weighted equally. This combination is of particular advantage for short data records because equal weighting amounts to pooling the interval sequences at all given levels thus resulting in a more consistant estimate than that obtained using a single level.

Let the random variable denoting the interval sequence at level A_l be Γ_l. Let Γ denote the pooled interval sequence. Approximating the mean η_l to be invariant with level ($\eta_l = \eta \ \forall a_l$) the variance σ of Γ can be shown to be related to the variances of the LC interval data at individual levels.

$$\sigma^2 = \frac{1}{N} \sum_{l=0}^{L-1} N_l \sigma_l^2. \tag{15}$$

where N_l is the number of intervals obtained at level a_l and N is the total number of intervals accumulated over all the L levels. The mean of Γ is equal to η. Eqn.(15) suggests that the variance of the pooled MLC interval data is a weighted sum of the variances of LC intervals at individual levels. Since the weights depend on the number of intervals at each level, one can interpret them as a measure of statistical consistancy at that level.

For infinite data, the weight N_l/N would reach an asymptotic value. In general, N_l/N depends on the amplitude distribution of the signal, i.e., $N_l/N \propto Prob(x > A_l)$.

Bandpass noise

For the case of $x_v(t)$, since the amplitude distriution is Gaussian, $N_l/N \propto (1 - erf(x))$ where $erf(x)$ is the error function. Because of the nature of the error function, N_l/N can be represented by $e^{-c_2\, i}$, [1] where $i\sqrt{E_v} = A_l$. Thus, the standard deviation σ_v for the MLC case can be simplified to

$$\sigma_v = \sigma_v(0) \sqrt{\sum_{l=0}^{L_v-1} e^{\sqrt{\pi/2b}(A_{max}/g)(2c_1-c_2)a_l}} \, . \tag{16}$$

where $\sigma_v(0)$ is the variance of the ZLC data and $L_v = \alpha\sqrt{E_v}.L/A_{max}$.

Sinusoid plus noise

The amplitude distribution of the sinusoid plus noise at high SNRs can be approximated to be uniform. Thus,

$$\begin{aligned} N_l/N &= 1/L_m, \quad l = 0,..L_m - 1 \\ &= 0, \quad l = L_m,..L-1 \end{aligned} \tag{17}$$

where $L_m = A_m L/A_{max}$ is the number of levels crossed by the simunoid. The stndard deviation of the pooled data is given by

$$\sigma_s = \sigma_s(0) \sqrt{\frac{1}{L_m} \sum_{l=0}^{L_m-1} \frac{1}{(1 - r_m^2 a_l^2)}} \tag{18}$$

where $\sigma_s(0)$ is the standard deviation for the ZLC data.

Detection threshold

The detection measure for the MLC estimator is also formulated as the ratio of the mean to standard deviation of Γ. Appying the MAP criterion, the detection threshold can be expressed as $(\eta_s/\sigma_s) > (\eta_v/\sigma_v)$. Substituting the pooled mean and variances, the SNR threshold for sinusoidal detection using the MLC estimator can be derived as follows:

$$\frac{A_m \pi}{g} > \frac{\sqrt{12}\omega_c}{\sqrt{\pi b}} \frac{\sqrt{\frac{1}{L_m}\sum_{l=0}^{L_m-1} \frac{1}{(1-r_m^2 a_l^2)}}}{\sqrt{\sum_{l=0}^{L_v-1} e^{\sqrt{\pi/2b}(A_{max}/g)(2c_1-c_2)a_l}}} \tag{19}$$

[1]Strictly, c_2 is also a function of the noise variance. However, the choice of c_2 for high SNR will lead to a conservative estimate of the noise threshold.

Numerical evaluation of (19) shows that the MLC analysis can provide a lower SNR threshold for sinusoid detection than the ZLC analysis.

Simulation Results

Simulation results presented in Fig. 5 show the sinusoid detection threshold (SNR in dB) for the ZLC and MLC analysis. The improvement in threshold using MLC analysis with 5 levels, distributed uniformly on a log scale, is found to be $\approx$ 7dB.

The performance of the MLC estimator for single sinusoid in noise can also be compared with the standard maximum likelihood (ML) method which is an optimum estimator. Results obtained from Monte-Carlo simulations show that the estimation performance of MLC is comparable to that of the ML estimator. Fig. 6 shows the ML and MLC estimates of a single frequency ($.25\pi$) at various local SNRs. It can be seen that the performance of both the estimators is similar at high SNR, whereas at low SNR the ML estimate has higher bias than the MLC estimate.

Conclusions

The analysis and simulation results presented in this paper establish the noise robustness of the MLC based spectral estimator. This method provides reliable multiple sinusoid estimation in noise provided the filters in the bank are narrow enough to pass only a single sinusoid at the output of the filter. This implies that the resolution of the MLC estimator is a function of the bandwidth of the filter. Narrow bandwidth filter banks call for long impulse response which requires more signal data.

References

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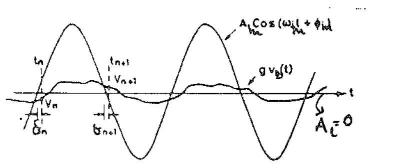

Fig. 1 Illustration of perturbation of level crossings of a sinusoidal signal due to additive bandpass noise.

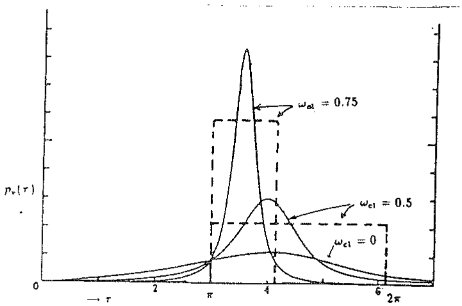

Fig. 2 Theoretical Approximation to the pdf, $p_v(\tau)$, of ideal bandpass noise [2]. Uniform pdf approximation to $p_v(\tau)$ used in this paper is shown by dashed lines.

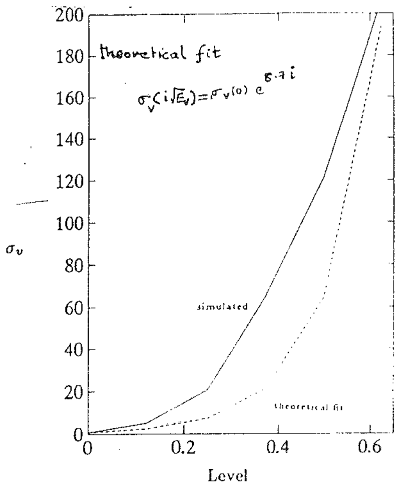

Fig. 3 σ_v Vs. level

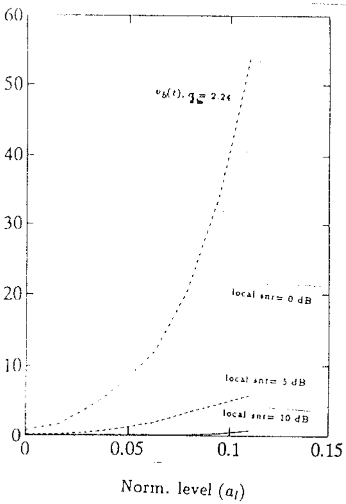

Fig. 4 σ Vs. level

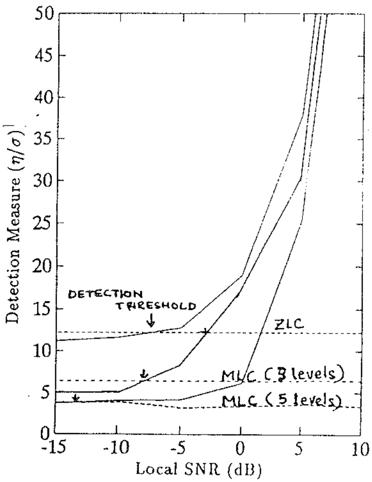

Fig. 5 Detection Threshold for MLC analysis

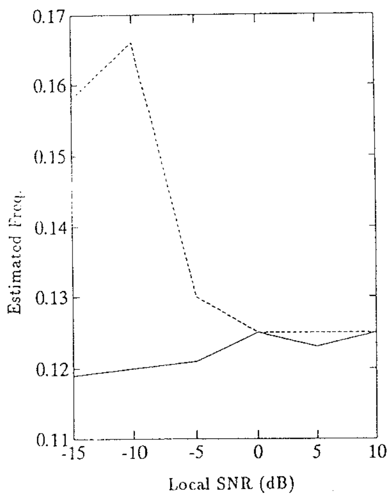

Fig. 6 Performance of MLC estimator