IMPROVED ALGORITHMS FOR VQ CODEWORD SEARCH AND THE DERIVATION OF BOUND FOR QUADRATIC METRIC USING PRINCIPAL COMPONENT TRANSFORM

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ABSTRACT - A new bound for quadratic metric using principal component transform is derived in this paper. A new fast search algorithm for quadratic metric is also proposed by storing the transformed codeword first, then the algorithm is executed by using the previous vector candidate, bound for quadratic metric and partial distortion search from the transformed input data. Experimental results demonstrate that this new algorithm compared with previous work (Pan et al., 1996a) will reduce the number of multiplications and the total number of mathematical operations for 1,024 codewords by more than 77% and 50%.

INTRODUCTION

Vector quantization (VQ) (Gray, 1984) has widely been used in speech coding, speech recognition, and image coding. One of the most serious problems in VQ is the search cost to find a minimum distortion codeword from a given codebook. Given one codeword C_t and the test vector X in the k-dimensional space, the distortion of the squared Euclidean metric can be expressed as follows:

$$D(X, C_t) = \sum_{i=1}^{k} (x^i - c_t^i)^2, \tag{1}$$

where $C_t = \{c_t^1, c_t^2, ..., c_t^k\}$ and $X = \{x^1, x^2, ..., x^k\}$.
Each distortion calculation requires k multiplications and $2k - 1$ additions. Therefore, it is necessary to perform $k2^{kr}$ multiplications, $(2k-1)2^{kr}$ additions, and $2^{kr} - 1$ comparisons for encoding each input vector. The computation complexity depends on codebook size and dimensions. It needs large codebook size and higher dimension for high performance in VQ encoding and recognition systems resulting in increased computation load during codeword search.

In order to reduce the search cost, the partial distortion search (PDS) algorithm (Bei & Gray, 1985) has been proposed. The PDS is a simple and efficient method which allows early termination of the distortion calculation between an input vector and a codeword by introducing a premature exit condition in the search process. Given the current minimum distortion,

$$D(X, C_t) = D_{min}, \tag{2}$$

$$if \qquad \sum_{i=1}^{s} (x^i - c_j^i)^2 \geq D_{min}, \tag{3}$$

$$then \qquad D(X, C_j) \geq D(X, C_t), \tag{4}$$

where $s \leq k$.
The efficiency of PDS derives from elimination of an unfinished distortion computation if its partial accumulated distortion is larger than the current minimum distortion. This will reduce computation to $(k-s)$ multiplications and $2(k-s)$ additions at the expense of s comparisons.

For speech data, the classification result of the present vector is usually the same as or close to the classified result of the previous vector. The nearest codeword of the previous vector can be used as the tentative match called previous vector candidate which is first proposed by (Pan, 1988; Chen & Pan, 1989). In vector quantization of images, data are first divided into subsequent blocks of size $k = M \times M$. The previous vector candidate has also been applied to image coding (Huang & Chen, 1990) by taking the advantage of high correlation between contiguous subimages. The previous vector candidate, Criterion 1 of the triangular inequality elimination and the partial distortion search were also applied to Manhattan (Chebyshev) metric for VQ image coding by (Nyeck et al., 1992).

The bound for Minkowski metric has been derived in the previous work (Pan et al., 1996a) as follows:

$$if \qquad \sum_{i=1}^{s} |x^i - c_j^i|^p \geq \sqrt[\frac{n}{p}]{h^{\frac{n}{p}-1} D_{min}}, \tag{5}$$

$$then \qquad \sum_{i=1}^{k} |x^i - c_j^i|^n \geq D_{min}, \tag{6}$$

where $s \leq h \leq k$ and $p \leq n$.
The improved absolute error inequality (IAEI) criterion (Pan et al., 1996b) is obtained by setting n=2 and p=1. Hence IAEI criterion is expressed as follows:

$$if \qquad \sum_{i=1}^{s} |x^i - c_j^i| \geq \sqrt{h D_{min}}, \tag{7}$$

$$then \qquad \sum_{i=1}^{k} (x^i - c_j^i)^2 \geq D_{min}, \tag{8}$$

where $s \leq h \leq k$.

BOUND FOR QUADRATIC METRIC

In the previous work (Pan et al., 1996a), a bound for quadratic metric has already been derived using triangular matrix technique. The main spirit is to split the covariance matrix to the product of the lower triangular matrix and the upper triangular matrix so that the quadratic metric is transformed to the Euclidean metric. A new bound for quadratic metric is also derived using The Karhunen-Loêve transform (KLT). KLT is also called the eigenvector transform, principal component transform and Hotelling transform. It is an optimal transform in a statistical sense under a variety of criteria. The KLT has the following properties (Elliott and Rao, 1982):

1. It is the best vector transform in the sense of decorrelating the sequence completely in the transform domain.
2. It packs the most energy (variance) to the low order elements.
3. It minimizes the mean squared error (MSE) between the original and reconstructed data for any specified bandwidth reduction or data compression.
4. It minimizes the total entropy of the data sequence.

Eigenvectors of the covariance matrix of a given sequence are the basis functions of the KLT. Assume P and Λ are the eigenvector and diagonal matrix of eigenvalues, respectively. The quadratic metric can be transformed to the Euclidean metric as follows:

$$D(X, C_m) = (X - C_m)^t W^{-1} (X - C_m)$$

$$= (X - C_m)^t \{P \Lambda P^t\}^{-1} (X - C_m)$$

$$= (X - C_m)^t \{P \begin{bmatrix} \lambda_1 & 0 & 0 & \dots & 0 \\ 0 & \lambda_2 & 0 & \dots & 0 \\ 0 & 0 & \lambda_3 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \lambda_k \end{bmatrix} P^t\}^{-1} (X - C_m)$$

$$= (X - C_m)^t \{P^{t^{-1}} \begin{bmatrix} \lambda_1 & 0 & 0 & \dots & 0 \\ 0 & \lambda_2 & 0 & \dots & 0 \\ 0 & 0 & \lambda_3 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \lambda_k \end{bmatrix}^{-1} P^{-1}\} (X - C_m)$$

$$= (X - C_m)^t \{P \begin{bmatrix} \frac{1}{\lambda_1} & 0 & 0 & \dots & 0 \\ 0 & \frac{1}{\lambda_2} & 0 & \dots & 0 \\ 0 & 0 & \frac{1}{\lambda_3} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \frac{1}{\lambda_k} \end{bmatrix} P^t\} (X - C_m)$$

$$= (X - C_m)^t \{P \begin{bmatrix} \frac{1}{\sqrt{\lambda_1}} & 0 & 0 & \dots & 0 \\ 0 & \frac{1}{\sqrt{\lambda_2}} & 0 & \dots & 0 \\ 0 & 0 & \frac{1}{\sqrt{\lambda_3}} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \frac{1}{\sqrt{\lambda_k}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{\lambda_1}} & 0 & 0 & \dots & 0 \\ 0 & \frac{1}{\sqrt{\lambda_2}} & 0 & \dots & 0 \\ 0 & 0 & \frac{1}{\sqrt{\lambda_3}} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \frac{1}{\sqrt{\lambda_k}} \end{bmatrix} P^t\} (X - C_m)$$

$$= (X - C_m)^t Q Q^t (X - C_m) = U^t U = \sum_{i=1}^{k} |u^i|^2, \qquad (9)$$

where

$$Q = P \begin{bmatrix} \frac{1}{\sqrt{\lambda_1}} & 0 & 0 & \dots & 0 \\ 0 & \frac{1}{\sqrt{\lambda_2}} & 0 & \dots & 0 \\ 0 & 0 & \frac{1}{\sqrt{\lambda_3}} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \frac{1}{\sqrt{\lambda_k}} \end{bmatrix},$$

$U = Q^t (X - C_m)$ and u^i is the element of the row matrix U.

Apply the IAEI to Eq. 9 and assume the current minimum distortion

$$D_{min} = D(X, C_m). \quad (10)$$

$$If \qquad \sum_{i=1}^{s} |u^i| \geq \sqrt{h D_{min}}, \quad (11)$$

$$then \qquad D(X, C_j) \geq D_{min} \quad (12)$$

where $s \leq h \leq k$.

After the transform of quadratic metric to Eq. 9 using KLT, another bound for quadratic metric is derived as shown in Eq. 10 to Eq. 12.

NEW CODEWORD SEARCH ON EUCLIDEAN METRIC

In the previous work (Pan et al., 1996a), the test materials consisted of two hundred words recorded from one male speaker. The speech was sampled at a rate of 16 kHz and 13-dimensional cepstrum coefficients with inverse variance weighting were computed over 20 ms-wide frames with a 5 ms frame shift. A total of 20,030 analyzed frames were used in the codeword searching experiments. A fast algorithm is implemented by using the minimum value of the maximum dimension-distortion as the tentative match, the PDS and the IAEI, setting h to 1, 4, 9 and 13, adapting and comparing Eq. 7 for s from 1 to 13 in the previous work. Another possible approach is to adapt s from 1 to 13 but only compare Eq. 7 at s = 1, 4, 9 and 13. As shown in Table 1, this approach will decrease the number of comparisons as well as the total number of operations at the expense of more additions. In terms of the total number of mathematical operations, this approach is a little better than the minimax method but drastically reduces the number of multiplications for 1024 codewords.

method	mul.($\times 10^3$)	cmp.($\times 10^3$)	add.($\times 10^3$)	sum($\times 10^3$)
Minimax	7,569	292,892	272,682	573,143
Minimax_AEI	2,133	305,783	285,573	593,489
IAEI_Euclidean	1,671	299,002	278,865	579,538
New_IAEI_Euclidean	1,671	291,360	279,969	573,000

Table 1: Computational complexity of codeword search for 1024 codewords on Euclidean metric

NEW CODEWORD SEARCH ON QUADRATIC METRIC

A modified method can be applied to previous fast algorithm (Pan et al., 1996a) by preprocessing $C_m^t L$ first, then XL can be operated outside the loop of the codeword search. This modified method is more efficient than previous one. Assume z_{mi} is the element of the vector $C_m^t L$, $1 \leq m \leq N$, $1 \leq i \leq k$. The modified algorithm is described as follows:

Step 1: Compute the nearest neighbour for the first frame X_1. For the other frame X_p, use the nearest neighbour of X_{p-1} (previous vector candidate) as a tentative match and so find the initial value of D_{min}.

Step 2: Calculate $X_p{}^t L = (y_1, y_2, ..., y_k)$.

Step 3: For every codeword C_j, calculate steps 3 to 7.

Step 4: For every dimension (i from 1 to k), calculate steps 4 to 6.

Step 5: Calculate $|E_j^t V_i| = \sum_{r=1}^{i} |y_r - z_{jr}|$.

Step 6: If $\sum_{m=1}^{i} |E_j^t V_m| \geq \sqrt{h D_{min}}$, $h \geq i$, then C_j will not be the nearest neighbour to the frame X_p, therefore go to step 3 for the next codeword.

Step 7: Calculate $|E_j^t V_i|^2$. If $\sum_{m=1}^{i} |E_j^t V_m|^2 \geq D_{min}$, then C_j will not be the nearest neighbour to the frame X_p, therefore go to step 3 for the next codeword.

Step 8: If $\sum_{m=1}^{k} |E_j^t V_m|^2 < D_{min}$, set $D_{min} = \sum_{m=1}^{k} |E_j^t V_m|^2$ and record C_j as the nearest neighbour to X_p.

The test materials for these experiments consisted of 99 words recorded from one male speaker. The speech was sampled at a rate of 16 kHz and 13-dimensional cepstrum coefficients were computed over 20 ms-wide frames with a 5 ms frame shift. The total number of frames is 9,391. Experiments were carried out to test the performance of the conventional exhaustive method, the fast codeword search algorithm without tentative match approach (i.e. with C_1 as the tentative candidate) of the previous work, the fast codeword search algorithm with quadratic metric of the previous work and the proposed new codeword search algorithm for 256, 512 and 1024 codewords. The conventional exhaustive method is referred to as "Conventional". The fast codeword search algorithm without tentative match approach and the fast codeword search algorithm of the previous work are referred to as "No-quadratic" and "Pre-quadratic", respectively. The proposed new algorithm is reerred to as "New-quadratic". The bounds for quadratic metric are separated into four sections ($h = 1, 4, 9, 13$). Experimental results is shown in Table 2, Table 3 and Table 4. In terms of the total number of mathematic operations, the modified version can reduce by more than 50 % computation complexity. No extra memory is needed if the same matrix W is used throughout. Hence the original codewords need not be stored, but can be replaced completely by the transformed codewords $C_m^t L$.

method	mul.($\times 10^3$)	cmp.($\times 10^3$)	add.($\times 10^3$)	sum($\times 10^3$)
Conventional	437,545	2,395	435,141	875,081
No-quadratic	82,691	26,354	92,776	201,821
Pre-quadratic	50,685	18,630	57,003	126,318
New-quadratic	10,926	18,630	27,957	57,513

Table 2: Computational complexity of codeword search for 256 codewords on quadratic metric

method	mul.($\times 10^3$)	cmp.($\times 10^3$)	add.($\times 10^3$)	sum($\times 10^3$)
Conventional	875,091	4,799	870,283	1,750,173
No-quadratic	142,364	47,619	160,012	349,995
Pre-quadratic	86,963	33,569	97,912	218,444
New-quadratic	18,837	33,569	49,433	101,839

Table 3: Computational complexity of codeword search for 512 codewords on quadratic metric

CONCLUSIONS

A new bound for quadratic metric using principal component transform is derived in this paper. An efficient codeword search algorithm for quadratic metric is also proposed. Experiments demonstrate that the proposed new algorithm will reduce the number of multiplications and the number of mathematical operations for 1024 codewords by more than 77% and 50%, respectively.

method	mul.($\times 10^3$)	cmp.($\times 10^3$)	add.($\times 10^3$)	sum($\times 10^3$)
Conventional	1,750,182	9,607	1,740,566	3,500,355
No-quadratic	246,729	86,226	277,687	610,642
Pre-quadratic	147,768	59,737	166,366	373,871
New-quadratic	32,555	59,737	86,882	179,174

Table 4: Computational complexity of codeword search for 1024 codeword on quadratic metric

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