FAST, STABLE SOLUTION OF THE TWO-DIMENSIONAL COCHLEAR MODEL

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ABSTRACT - The two-dimensional model of the cochlea is solved in the time-domain. The use of the bi-linear transformation leads to faster and stable responses over previous methods. Plots of basilar membrane velocity are presented for an active model both as a function of distance and frequency.

INTRODUCTION

One way of improving our understanding of speech perception is to model the functions involved in the act of hearing. A well-known model of the auditory system is derived from the two-dimensional mathematical description of the cochlear chamber. It represents the best compromise between the inaccuracies of a one-dimensional model and the computational complexities of a three-dimensional representation.

Its use in the time-domain ,however, has been restricted by the inability of known solutions to converge unless very small sampling times are adopted. By using the bi-linear transformation a stable and a faster solution has been developed.

DESCRIPTION OF MODEL

Solution technique

The mathematical derivation of the model from Viergever (1980) yields the boundary-value problem posed by Fig. 1. The problem of Fig. 1 is the solution of Laplace's equation subject to boundary conditions. One of the easiest and most flexible techniques for solving this is the finite difference method (Isaacson & Keller,1966).The method involves representing the pressure differences by a set of discrete points in the x and y directions.The equations are also replaced by second-order accurate approximations and the problem is transformed to that of solving a matrix.

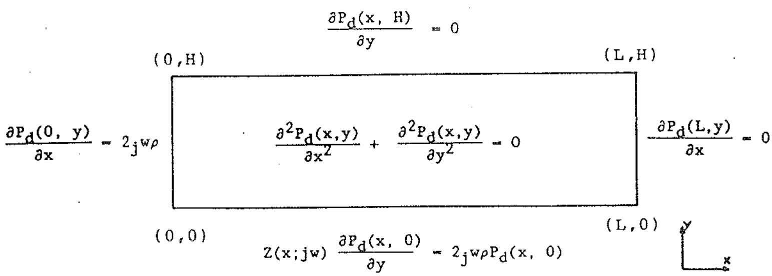

Figure 1. Boundary value problem of the two dimensional cochlear model

Both the key issue and the main difficulty of the solution is the Basilar Membrane(BM) condition and the complex-valued BM impedance Z(x;s). Although a frequency-domain solution has been successfully tried ,a time-domain solution,which is more practical for speech processing, requires solution at each time-step and can lead to long computation times. Furthermore, the discretisation in the time-domain must not lead to instability.

The stapes condition can be discretised using the forward difference approximation:

$$s = \frac{1 - z^{-1}}{T} \qquad s^2 = \frac{1 - 2z^{-1} + z^{-2}}{T^2} \qquad (1)$$

The expression 2jwp/Z(x;s) for the BM pressure gradient exhibits a high-pass filter characteristic and this must be preserved. The bi-linear transformations was chosen as the most accurate and easiest way to discretise the BM boundary equation.The transformation is defined by:

$$s = \frac{2}{T}\frac{(1 - z^{-1})}{(1 + z^{-1})} \qquad (2)$$

Applying this for the BM condition where Z(s) = Z(x;s) (i.e. dependence on x is implied) gives:

$$\frac{\partial P_d(x,0)}{\partial y} = 2\rho \frac{s}{Z(s)} P_d(x,0) = 2\rho H(z) P_d(x,0)$$

$$= 2\rho \left[\frac{N_1(z) + N_2(z^{-1} .. z^{-n})}{D_1(z) + D_2(z^{-1} .. z^{-n})}\right] P_d(x,0)$$

$$\therefore \frac{\partial P_d(x,0;n)}{\partial y} = 2\rho \frac{N_1(z)}{D_1(z)} P_d(x,0;n)$$

$$+ \frac{2\rho N_2(z^{-1} .. z^{-n}) P_d(x,0;n) - D_2(z^{-1} .. z^{-n}) \frac{\partial P_d(x,0;n)}{\partial y}}{D_1(z)}$$

$$= M(x) P_d(x,0;n) + C(x) \qquad (3)$$

The solution vector is defined by:

$$\tilde{P} = \begin{bmatrix} P_1 \\ P_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ P_J \end{bmatrix} JK \quad \text{where} \quad \bar{P}_j = \begin{bmatrix} P_{j1} \\ P_{j2} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ P_{jK} \end{bmatrix} K \qquad (4)$$

Hence the resultant matrix and forcing vector are of the form:

$$A = \begin{bmatrix} D_1 & 2\theta_x I_k & & & \\ \theta_x I_k & D_2 & \theta_x I_k & & \\ & \cdot & \cdot & \cdot & \\ & & \cdot & \cdot & \cdot \\ & & & 2\theta_x I_k & D_J \end{bmatrix} JK \qquad F = \begin{bmatrix} \tilde{f}_1 \\ \tilde{f}_2 \\ \cdot \\ \cdot \\ f_J \end{bmatrix} JK$$

where

$$\tilde{f}_1 = \begin{bmatrix} 2a_s\delta_x\theta_x + 2\delta_y\theta_y C(x) \\ 2a_s\delta_x\theta_x \\ \cdot \\ \cdot \\ \cdot \\ 2a_s\delta_x\theta_x \end{bmatrix} K \qquad \tilde{f}_j = \begin{bmatrix} 2\delta_y\theta_y C(x) \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} K$$

$$D_j = \begin{bmatrix} bmf(x) & 2\theta_y & & & \\ \theta_y & -1 & \theta_y & & \\ & \cdot & \cdot & \cdot & \\ & & \cdot & \cdot & \cdot \\ & & & 2\theta_y & -1 \end{bmatrix} K$$

$$I_k = K \times K \text{ identity matrix} \qquad (5)$$

$$bmf(x) = -(1 + 2\delta_y\theta_y M(x))$$

At each time=step the following matrix equation has to be solved:

$$AP = F \qquad (6)$$

The Gaussian block elimination technique (Isaacson & Keller,1966)is used to solve the matrix. Furthermore, since the matrix is time-invariant the forward substitution phase of the procedure is initially carried out and stored. Hence only modification of F and back-substitution is required at each time-step.

Basilar Membrane impedance function

The form of the impedance function Z(x;s) can be determined from consideration of the physical representation of the BM and surrounding structures. Various interpretations have been proposed in order to reconcile both the description with the physiology and the model response with the highly tuned characteristic present in the BM measurement data of Fig. 2.

Perhaps the most accepted proposal came from Neely (1980) which assumes the presence of an active component to account for the sharp tuning. In this model the BM consists of mass,damping and stiffness as in the simple,passive case. In addition the concept of a negative damping or resistance which couples the BM with hair cell has been introduced. In this way energy can be transferred back to the cochlear chamber causing the BM motion to be amplified. The following form for Z(x;s) and component values are taken from Neely (1983) and used for the results presented here:

$$Z(x;s) = sM_1(x) + R_1(x) + K_1(x)/s + \frac{-R_3(x)^2}{sM_2(x) + R_2(x) + K_2(x)/s} \quad (7)$$

RESULTS

The following parameters were used: length of cochlea L = 2.55cm, height of cochlea H = 0.1cm and fluid density p = 1.0 g/cm3. The pressure differences were represented at 409 equispaced points along the x direction and 8 points in the y direction. Typical computation times on an IBM-PC fitted with a DSI-32 coprocessor card was 2.5 min for 100 time-steps of the time-domain solution.

The time-domain model was solved excited by a sinusoidal input of 5kHz. Two cases were considered with model run times NT=40 ms, where N is the number of solution time-steps and T is the sampling time, and sampling times of 2 and 20 microseconds respectively. These are shown in Fig. 3 together with the solution for the equivalent steady-state response (from a frequency-domain model). The plots represent the averaged absolute output over the final 500 time-steps of the cochlear state. In all cases the responses in the region surrounding the peak are identical. Motion of the BM in the time-domain beyond this region is still dominated by transients and this explains the deviation with the steady-state.

The results indicate that both the proposed time-domain model response is valid, when compared to the steady-state, and that sampling frequencies (1/T) no greater than ten times the highest input excitation frequency can be used. This represents a considerable improvement over known solution methods (Allen & Sondhi, 1979; Neely, 1981; Neely, 1983) which yield unstable responses unless the sampling frequency is at least ninety times the highest input frequency. Furthermore, with the bi-linear transformation longer sampling times lead to warping of the response but not to instability. This is significant as there is another source of instability arising from the spatial discretisation of the solution space. An unstable response was evident when the number of points used to discretise the x-direction was reduced to 246. Hence any unstable responses which arise can be attributed to errors in the spatial discretisation.

A better representation of the model behaviour was obtained by applying an impulse as input and collecting the output response at selected positions. The ratio of the Fourier transform of the output over that of the input gave the response vs. frequency as shown in Fig. 4 for NT=20.48 ms and a sampling time 10 microseconds.In this way adjusting the parameter values can be used to match the response to BM measurement data (Sellick,Patuzzi & Johnstone, 1982).

CONCLUSION

This technique represents a fast and stable solution of the two-dimensional cochlear model of Fig.1.. It can be used to both process speech and examine the behaviour of various expressions for the BM impedance function without fear of ambiguous interpretation of the results.

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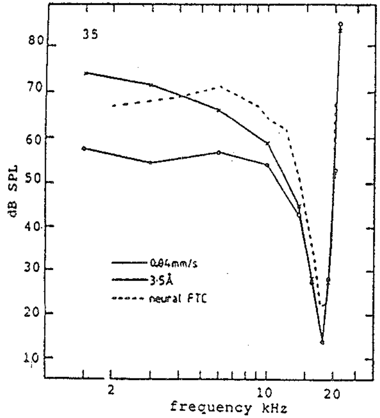

Figure 2. Measurements of basilar membrane isovelocity and isodisplacement curves compared with the Frequency Threshold Curve (FTC) (Reproduced from Sellick,Patuzzi & Johnstone, 1982)

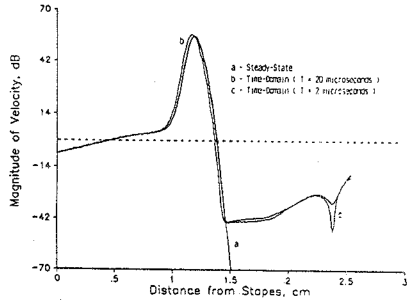

Figure 3. Comparison of the steady-state and time-domain cases for the active model. Solution was at 5 kHz with $R_3(x) = 1.0$ and NT = 0.04.

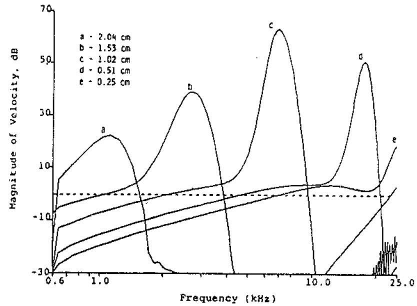

Figure 4. Basilar membrane velocity curves for the active, time domain model as a function of frequency (T= 10 microseconds). Distance along BM partition is as shown. Solution was at $R_3(x) = 1.0$ and N = 2048.