A MODIFIED RECURSIVE SOLUTION FOR THE LINEAR PREDICTIVE CODING OF SPEECH

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ABSTRACT In this paper a new algorithm is proposed to update recursively the parameter vector of a linear predictor for speech coding. The proposed method is based on modified Cholesky (LD) factorization of an augmented covariance matrix. In essence the algorithm suggested is a modified version of the method presented by (Ljung and Soderstrom, 1983) with distinct advantages. Firstly, the predictor parameter vector is directly obtained from the first column of the L factor, hence eliminating a computational step. Secondly, the given algorithm can readily and simply be implemented using standard systolic array structures (Jover, and Kailath, 1986).

INTRODUCTION

A popular linear predictive coding (LPC) of speech is based on estimating the parameters of the model described by (Rabiner and Schafer, 1978) as:

$$\frac{S(z)}{U(z)} = \frac{G}{1 - \sum_{k=1}^{P} a_k z^{-k}} \qquad (1)$$

Where G is the gain, a_k are the filter coefficients, while S(z) and U(z) represent the speech samples, and input excitation respectively. Conversion of equation (1) to its time domain expression can be stated as:

$$s(t) = \sum_{k=1}^{P} a_k \, s(t-k) + G\,u(t) \qquad (2a)$$

or equivalently

$$s(t) = \phi^T(t)\ \Theta(t) \qquad (2b)$$

where $\phi^T(t) = [s(t-1)\ s(t-2)\ \ldots\ s(t-p)\ u(t)]$

and $\Theta^T(t) = [a_1\ a_2\ \ldots\ a_p\ G]$

The least squares estimate of $\Theta(t)$ is given by (Åström and Wittenmark, 1984)

$$\hat{\Theta}(t) = R^{-1}(t)\, B(t) \qquad (3)$$

where $$R(t) = \sum_{i=t-L}^{t} \phi(i)\, \phi^T(i) \tag{4}$$

$$B(t) = \sum_{i=t-L}^{t} \phi(i)\, s(i) \tag{5}$$

Note that $\hat{\theta}(t)$ is the estimate of $\theta(t)$ based on L + 1 data samples, and T denotes the transpose. For further analysis we define an augmented information matrix M(t) given by:

$$M(t) = \begin{bmatrix} s^2(t) & B^T(t) \\ B(t) & R(t) \end{bmatrix} \tag{6}$$

Now consider the modified Cholseky factorization of $M^{-1}(t)$, that is:

$$M^{-1}(t) = L(t)\, D(t)\, L^T(t) \tag{7}$$

Where L is the unit lower triangular matrix, and D is the diagonal matrix. By exploiting some properties of the above factorization the following theorem applies:

<u>Theorem</u>: The least squares estimate $\theta(t)$ which satisfies equation (3) is given by:

$$\hat{\theta}^T(t) = -\,[L_{2,1}(t)\ L_{3,1}(t)\ \ldots.\ L_{p+1,1}(t)] \tag{8}$$

where $L_{i,j}(t)$ is the (i,j) element of matrix L(t).

<u>Proof</u>: See (Magdy et al, 1985)

On the basis of the above theorem, the rest of the paper will descibe an algorithm to update the estimate $\hat{\theta}(t)$.

PREDICTOR PARAMETER UPDATING

Close examination of equation (4) reveals that:

$$R(t) = R(t-1) + \phi(t)\, \phi^T(t) \tag{9}$$

In a similar manner from equation (6) we can write:

$$M(t) = M(t-1) + x(t)\, x^t(t) \tag{10}$$

For $x^T(t) = [s(t)\ \phi^T(t)]$

Now inverting both sides of equation (10) and using (7), we obtain

$$L(t)\, D(t)\, L^T(t) = [M(t-1) + x(t)\, x^T(t)]^{-1}$$

Factorizing M(t-1) in a similar way gives

$$L(t)\, D(t)\, L(t) = [L^{-T}(t-1)\, D^{-1}(t-1)\, L(t-1) + x(t)\, x^T(t)]^{-1}$$

$$= L(t)\, [D^{-1}(t-1) + hh^T]^{-1}\, L^T(t) \tag{11}$$

where $h = L^T(t-1)\ x(t)$

Applying the matrix inversion lemma on the square bracketed term of (11) we obtain

$$L(t)\ D(T)\ L^T(t) = L(t-D) \left[D(t-1) - \frac{D(t-1)\ hh^T\ D(t-1)}{1 + h^T D\ (t-1)\ h} \right] L^T(t-1) \qquad (12)$$

Equation (12) is used to update the factors L and D of the augmented matrix M^{-1}. The bracketed quantity on the RHS of equation (12) can be factored as

$$D(t-1) - \frac{D(t-1)\ h\ h^T\ D(t-1)}{1 + h^T\ D(t-1)h} = H\ D(t)\ H^T \qquad (13)$$

Where H is a lower unit triangular matrix. Exploiting the fact that the multiplication of two lower triangular matrices produces a lower triangular matrix, then

$$L(t) = L\ (t-1)\ H \qquad (14)$$

Since the predictor parameter vector $\hat{\Theta}(t)$ is included in the first column of L(t) then the algorithm to update $\hat{\Theta}(t)$ reduces to two basic steps. Firstly, performing the factorization of (13), and secondly obtaining L(t) by using equation (14).

THE ALGORITHM

In order to efficiently perform the two steps suggested in section 2 (i.e. to update $\hat{\Theta}(t)$) the following algorithm is adequate:

Initialization: $\alpha(p+2) = 1$

Main loop : For J = P+1,P,P-1, ... 1 do

$$h(J) = x(J) + \sum_{k=j+1}^{p+1} L(K,J).x(K)$$

$$g(J) = D(J).h(J)$$

$$\alpha(J) = \alpha(J+1) + g(J).h(J)$$

$$D(J) = (\alpha(J+1)/\alpha(J).D(J)$$

Inner loop: FOR I = J+1,J+2, ...P+1

SKIP when J = P+1

$B = L(I,J)$

$L(I,J) = U(I,J) - h(J).g(I)/\alpha(J+1)$

$g(I) = g(I) + B.g(J)$

END

END

CONCLUSION

The algorithm presented in this paper can be considered as an extension to the method presented by (Ljung and Soderstrom, 1983) to update Kalman filter gain. The modification is due to the application of the theorem referred to in section 1 such that the predictor parameter vector is directly obtained as the first column of the L-factor. This results in the elimination of a computational step. In addition the straight-forward implementation of the suggested algorithm in standard systolic array structures is of significant importance.

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