A COMPUTATIONAL MODEL OF THE BASILAR MEMBRANE

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ABSTRACT - A digital filter simulation of basilar membrane vibration is described, in which the cochlea is represented as a cascade of 128 digital filters. The parameters of each filter are derived from the mechanical characteristics of the membrane at corresponding points. Using the model it is possible to simulate at each point, the waveform of the sound pressure in the cochlear fluid, as well as the deflection of the membrane itself. Thus it is possible to calculate the frequency response of the membrane at every point relative to input at the stapes. In addition the deflection of the membrane along its length, at any instant in time can be obtained. The significant advantage of this model is its relatively rapid computation time.

INTRODUCTION

The basilar membrane (BM) varies in width and stiffness along its length. At the basal end it is narrow and stiff, growing wider and more flexible towards the apex. The maximum membrane displacement will occur near the stapes for high frequencies, and nearer the apex for low frequencies. Thus a low frequency vibration travels a long distance down the membrane, whilst a high frequency travels only a short distance.

The wave motion along the membrane is governed by its mechanical properties and those of the surrounding fluid. A number of mathematical models of cochlear dynamics have been reported, for example Zweig (1976), Nilsson and Moller (1977), and Dallos (1973). However, there exist only a few practical cochlea models, for example Lyon (1982), and Dolmazon et al (1978). The practical model of the cochlea presented here is a modification of the linear, one-dimensional transmission-line, proposed by Schroeder (1973).

DEVELOPMENT OF THE MODEL

A simplified electrical model of a basilar membrane section is shown in Figure 1. The parameters M,L,C and R are the per unit length series inductance, and shunt inductance/capacitance/resistance, respectively. The length of an elemental section is dx, and the membrane is considered to be a cascade of N such sections, with parameters M,L,C and R dependent on the distance x, of the section along the membrane.

In order to make the computational task more tractable, it is convenient to isolate each section from its neighbours. This isolation can be done without approximation if the interaction between sections is modelled by loading each section with Zt, the input impedance into the line at that particular point. A considerable simplification occurs if Zt is approximated by a parallel connectin of an inductance Mt, and a resistance Rt. (Dallos 1973). This arrangement is shown in Fig. 2, where the isolation is provided by the unity gain voltage amplifier. The voltage (pressure) transfer function can then be shown to be of the form

$$\frac{Vo(s)}{Vi(s)} = K\left(\frac{a}{s + a}\right)\left(\frac{Wp}{s + sBp + Wp}\right)\left(\frac{s + sBz + Wz}{Wz}\right) \quad .. (1)$$

low-pass filter — resonant pole — resonant zero

Where, s is the complex frequency variable, and K is an attenuation factor; a is the low-pass filter pole-frequency; Wp is the resonant pole-frequency and Bp is its bandwidth, and Wz is the resonant zero-frequency and Bz is its bandwidth.

Since for each section, Wp is always less than Wz, the over-all magnitude response will be low-pass, the pole/zero pair giving a steep cut-off. The filtering action of any particular section will thus have three consequences; frequencies below the pole frequency Wp will be transmitted with loss K (approx unity), frequencies near the pole frequency will resonate, and frequencies at and about the zero frequency Wz will be attenuated quite severely.

If x is the distance of a point on the basilar membrane from the stapes, then the resonant frequency fp, is given by Dallos (1973), as

$$fp(x) = 16000(10\text{\^{}}{-0.667x})\ \text{Hz} \quad .. (2)$$

where x is in centimeters. The maximum frequency which will excite the membrane is 16000 Hz, (x=0), basal end), and the resonant frequency at the apex (x=3.5 cm) is about 70 Hz.

The actual displacement of the BM is analogous to charge on the capacitor, and is proportional to the voltage Vm in Fig. 2. The displacement transfer function Vm/Vi, is given by

$$\frac{Vm(s)}{Vi(s)} = \left(\frac{a}{s + a}\right)\left(\frac{Wp}{s + sBp + Wp}\right) \quad .. (3)$$

This is a low-pass filter identical to that of the pressure transfer function (equation 1), but without the resonant zero. The cut-off of Vm/Vi is thus less steep than Vo/Vi.

FILTER PARAMETERS

If appropriate values of the filter parameters K,a,Wp,Bp,Wz,Bz are used in each filter section, the response of the whole BM model can be quite realistic. Unfortunately, the filter parameters cannot be obtained explicitly in terms of network element values M,R,L,C,Mt,Rt. Furthermore, the element values themselves are not all known accurately, being merely electrical analogues of the cochlea's mechanical properties. By analysing the network of Fig. 2, the following relationships can be obtained.

$$K = Mt/(Mt + M) \quad .. (4)$$

$$Wz = 1/(LC) \quad .. (5)$$

$$Bz = R/L \quad .. (6)$$

$$Bp + a = Rt/L + Rt/Lt + R/L \quad .. (7)$$

$$Wp + aBp = Rt.R/Lt.L + 1/LC \quad .. (8)$$

$$aWp = Rt/Lt.LC \quad .. (9)$$

where Lt = Mt.M/(Mt + M). Equations 7, 8 and 9 are not linear, thus an analytic solution is not possible. It is possible however, to use the equations to put constraints on the values of a, Wp and Bp. But before doing so it is convenient to define two ratios, r = Wz/Wp and q = a/Wz. The Q-factors of the resonant pole and zero are given by Qp = Wp/Bp and Qz = Wz/Bz. Equations 7, 8 and 9 can now be written in terms of r, q and the Q-factors.

$$(r - 1)(q - qr/Qp + r)/q = Lt/L \quad .. (10)$$

$$(r - 1)(q - q/Qz + 1)/q = Lt/L \quad .. (11)$$

$$r/Qp - 1/Qz = (r - 1)/q \quad .. (12)$$

It is known that the ratio Lt/L is small, and assuming it to be zero gives the result r = 1, that is, the pole and zero frequencies are equal. This solution is similar to the cochlea model of Lyon (1982). By allowing r > 1, it is possible to obtain responses more compatible with practical measurements. The following range of values have been found useful.

$10 < Qz < \infty$; $1.01 < r < 1.4$; $1 < Qp < 20$; $1.00 < q < 3.0$

DIGITAL FILTER SIMULATION

A digital filter model of the BM can be derived by transforming the equations 1 and 3 to sampled data form. Since the impulse response of the membrane is important, the impulse-invariant transform has been chosen for this purpose, (see Rabiner and Gold 1975). The form of the digital filter functions are as follows. The pressure transfer function is

$$\frac{Vo(Z)}{Vi(Z)} = K\left(\frac{1 - ao}{1 - aoZ}\right)\left(\frac{1 + b1 + b2}{1 - b1Z + b2Z}\right)\left(\frac{1 - a1Z + a2Z}{1 - a1 + a2}\right) \quad .. (13)$$

The displacement transfer function is

$$\frac{Vo(Z)}{Vi(Z)} = K\left(\frac{1 - ao}{1 - aoZ}\right)\left(\frac{(1 - b1 + b2)Z}{1 - b1Z + b2Z}\right) \quad .. (14)$$

It is clear from these equations that the displacement function is contained in the pressure transfer function. This leads to a simplification when the filter functions are converted into the filter structures shown in Fig. 3.

RESULTS

Figure 4 shows a family of displacement frequency responses at points 1/4, 3/8, 1/2 and 5/8 along the membrane, from the base. To a first approximation the cut-off rate of these curves is controlled by Qz, the Q-factor of the resonant zero. The peak height and the sharpness of the peak are controlled by Qp, the Q-factor of the resonant pole. The parameters of these curves have been adjusted to give shapes similar to measured responses, and this matching procedure was used to fine-tune the BM model. The BM displacement at any particular point can also be plotted as a function of time, and it is easy to obtain the response of the BM to an impulse at the stapes. The displacement at a particular instant in time can also be depicted as a function of distance along the BM. Fig. 5 shows the membrane 4 cycles after the onset of a 2 kHz sinewave. By superimposing several such displacement patterns, a "displacement envelope" is obtained.

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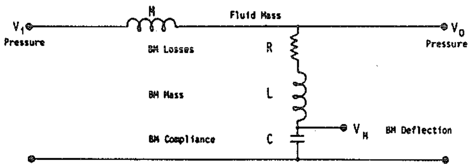


Fig 1. Elemental Section of BM

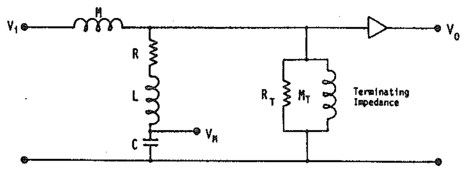


Fig 2. Isolated BM Section

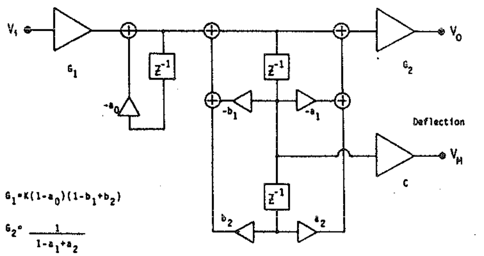
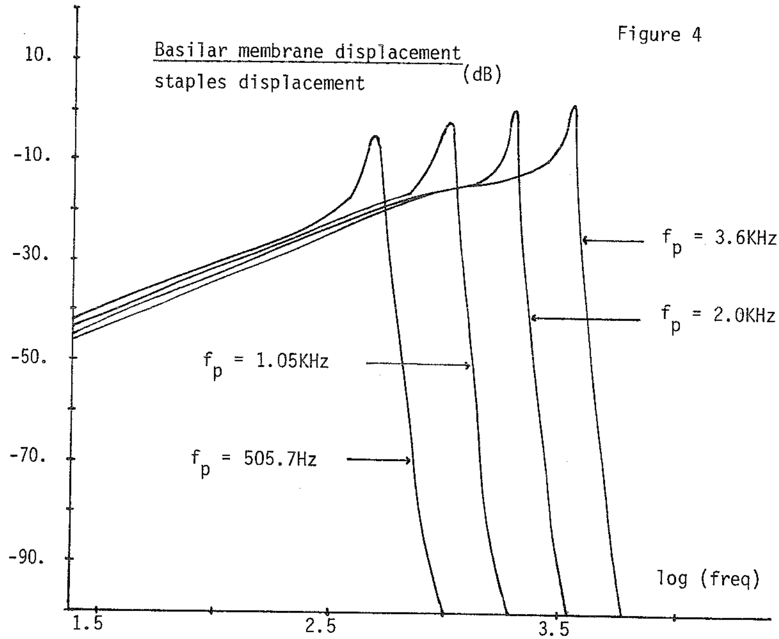
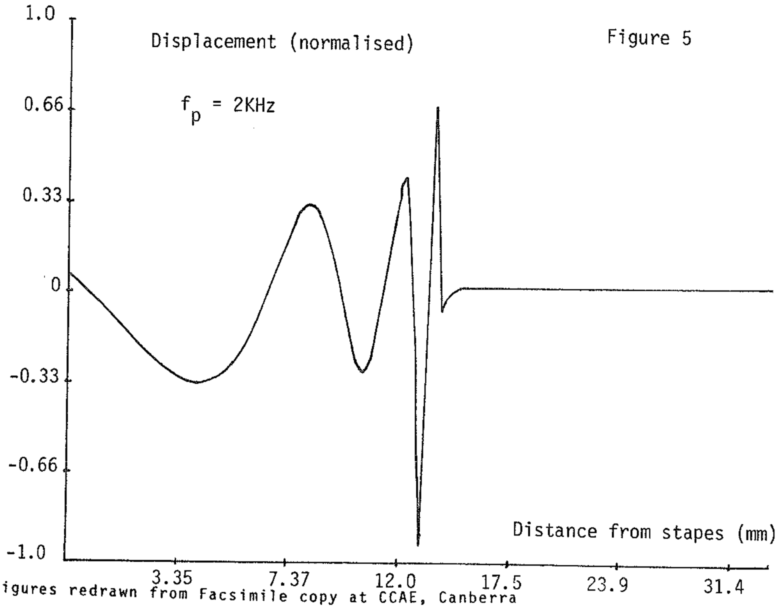


Fig 3. Digital Filter BM Section





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