A NEW APPROACH IN DESIGNING AN ADAPTIVE LATTICE PREDICTOR FOR NONLINEAR AND NONSTATIONARY SPEECH SIGNALS IN ADPCM USING LYAPUNOV THEORY

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ABSTRACT: In this paper, we present a computationally efficient adaptive lattice-ladder predictor for adaptive prediction of nonstationary speech signals in ADPCM. The important advantage of the proposed predictor is capable of adaptive predicting the signal and its algorithm does not require a priori knowledge of time dependent among the input data. The lattice reflection coefficients and the ladder weights are adaptively adjusted by algorithms that are designed using Lyapunov theory. The proposed scheme possesses distinct advantages of stability and speed of convergence over linear adaptive LMS or RLS lattice predictors in ADPCM. The theoretical derivation of the lattice predictor is further supported by simulation examples for speech signals.

1. INTRODUCTION

Many of the physical signals encountered in practice exhibits two distinct characteristics: nonlinearity and nonstationary. Consider, for example, the important case of speech signals. It is well known that the use of prediction plays a key role in the modeling and coding of speech signals [1]. The production of a speech signal is known to be resulted of a dynamic process that is both nonlinear and nonstationary. To deal with the nonstationary nature of speech signals, the customary practice is to invoke the use of adaptive predicting. One of the application of adaptive prediction is adaptive predictive coding. Predictive coding systems have been commonly used for encoding of speech, image and video signals. Among the adaptive predictive coding techniques, adaptive DPCM (*Differential Pulse Code Modulation*) [1],[2] is one of the most widely developed for predicting coding. The primary function of the adaptive predictor is to compute the estimate signal from the quantized difference signal and past values of the reconstructed signal. The difference between the input and the estimated signal is quantized and is then used for storage or transmission.

In the last decade there has been a considerable growth in Least Mean Square (LMS), Recursive Least Squares (RLS) algorithms and their modifications in adaptive linear predicting systems. The LMS lies in its computational simplicity but it is highly dependent on the autocorrelation function associated with the input signals and suffers slow convergence. Conversely, RLS exhibits consistent convergence properties but it is computationally expense to implement even with the availability of the fast algorithm and it exhibits unstable performance [3]. Methods of avoiding instability have been proposed in [4]-[5] but the stability problem of the adaptive filters have not been solved if there are some bounded input disturbances. The two main filter realizations that have been studied are the *transversal* and *lattice* filters. The lattice filter has become popular in the mid-1970's, primarily in the area of speech processing [1],[6]. One might wonder why the lattice structure is useful in light of the fact that its complexity is clearly greater than that of a transversal filter. One of the main attractions of lattice filters is that lattice filters tend to be more robust than transversal filters with respect to quantization and round-off noise [1]-[6]. Another advantage is that lattice filter is a convenient realization to use when the correct filter order is not known *a priori.* Furthermore the appeal of the lattice architecture is the regularity and modularity of the architecture.

In this paper, we present a computationally efficient adaptive lattice-ladder predictor for adaptive prediction of nonstationary speech signals in ADPCM. The important advantage of the proposed predictor is capable of adaptive predicting the signals and its algorithm does not require a priori knowledge of time dependent among the input data. The lattice reflection coefficients and the ladder weights are adaptively adjusted by algorithms that are designed using Lyapunov theory. A Lyapunov function instead of the cost function is defined for prediction error between the actual and estimate signals. Simultaneously, the prediction error convergence asymptotically rather than the minimization of the mean square error is desired to achieve. The stochastic properties of the signals are not required and the stability of prediction error is guaranteed by the Lyapunov Stability Theory. On the other hand, a new adaptive algorithm using Lyapunov theory is also designed to update reflection coefficients so that the forward prediction errors in the lattice structure can converge asymptotically. The proposed scheme possesses distinct advantages of stability and speed of convergence over linear adaptive LMS or RLS lattice predictors in ADPCM. The theoretical derivation of the lattice predictor is further supported by simulation examples for speech signals.

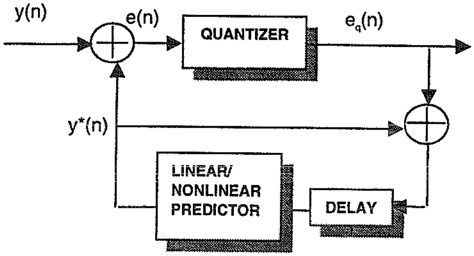

Figure. 1: Block diagram of a predictive coding system

2. SYSTEM DESCRIPTION

A block diagram of the ADPCM system is shown in Figure 1. In the system, the predictor generates a predicted value $y^*(n)$ of the current input sample $y(n)$, and the quantized difference between these two, $e_q(n)$, is transmitted to the receiver. There are two ways to implement a linear predictor: transversal structure and lattice structure. The later is the one we consider in this paper. Figure 2 shows the lattice-ladder filter predictor for ADPCM. In a lattice filter, the forward and backward prediction errors are updated by the recursions

$$f_i(n) = f_{i-1}(n) + k_i^{\,b}(n)b_{i-1}(n-1) \tag{2.1}$$

$$b_i(n) = b_{i-1}(n-1) + k_i^{\,f}(n)f_{i-1}(n) \tag{2.2}$$

where k_n^f and k_n^b are the nth-order reflection coefficients for the forward and backward predictors. It is usually assumed that $k_i^f(n) = k_i^b(n) = k_i(n)$. In transversal structure, those algorithms use the mean squared error criterion or least square criterion for determining the optimum set of predictor coefficients. In an adaptive lattice filter, the objective is to find the optimum set of reflected coefficients that minimizes the mean squared prediction error of the input process. There are different methods of updating the coefficients $k_i^f(n)$ and $k_i^b(n)$. The simple gradient algorithm is the least mean square (LMS) lattice algorithm [7]. Another lattice algorithm that converges faster than the gradient algorithm is the least squares (LS) algorithm [8]. The LS lattice algorithm is an exact solution to the best reflection coefficients that minimize the sum of the exponentially weighted squared prediction errors.

The output of the lattice-ladder predictor is a linear combination of the backward prediction errors, b_i. It can be represented as

$$y^*(n) = \sum_{i=1}^{P} v_i b_i(n) \tag{2.3}$$

or in vector notation by

$$y^*(n) = V^T(n)B(n) \tag{2.4}$$

where $V(n)$ is the vector of ladder coefficients and $B(n)$ is the vector of the backward prediction error, b_i.

$$B(n) = [b_1(n), b_2(n), b_3(n), \ldots, b_P(n)]^T$$

An adaptive algorithm is also used to update the ladder coefficients. The predictor uses *P* past samples of a signal to estimate the value of the current sample *x(n)*. If we denote the estimate of *y(n)* by *y*(n)*, then the difference or the prediction error is

$$e(n) = y^*(n) - y(n) \qquad (2.5)$$

which is quantized and then used for storage and transmission. It is obvious that the bit rate required for representing the difference *e(n)* is lower than that for the original signal *x(n)*. As the accuracy of the predictor increases, the variance of the difference will decrease coding involve designing and implementing the predictor.

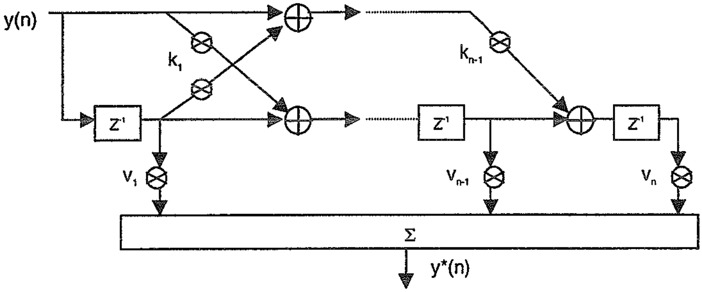

Figure. 2: Lattice filter predictor for ADPCM

3. PROPOSED ALGORITHM

Authors in [9] have proposed Lyapunov theory-based adaptive filtering (LAF). The design adaptive filter is based on transversal FIR structure and the adaptive algorithm is the modification of recursive least squares (RLS) algorithm using Lyapunov stability theory. The LAF method [11] is independent of the stochastic properties of the signals. Based on the observations and a collection of desired response, the filter parameters are updated in the Lyapunov sense so that the error between the desired response and the filter output can asymptotically converge to zero. This concept and recent work of Lyapunov adaptive filtering has lead to the development of adaptive lattice filter [7],[8], with the structure shown in Figure 2. This lattice filter can then be used in ADPCM as an adaptive lattice predictor.

It is necessary to derive the suitable learning algorithms for the reflection coefficients and the ladder coefficient updates. Most of the exiting learning algorithm is based on the gradient of the cost function which is a measure of the sum of square errors. The proposed learning algorithm operates in different principle. A Lyapunov is used instead of the cost function.

3.1 Ladder Weights Adaptation Algorithm

The ladder weights or coefficients can be adjusted as follow

$$V(n) = V(n-1) + g(n)\alpha(n) \qquad (3.1)$$

where *g(n)* is the adaptation gain and $\alpha(n)$ is the a priori estimation error defined as

$$\alpha(n) = y(n) - V^T(n-1)B(n) \qquad (3.2)$$

The adaptation gain *g(n)* in (3.1) is adaptively adjusted based on Lyapunov stability theory so that the error *e(n)* can asymptotically converge to zero.

$$g(n) = \frac{B(n)}{\| B(n) \|^2}\left(1 - \beta \frac{| e(n-1) |}{| \alpha(n) |}\right) \qquad (3.3)$$

To prevent singularities, the expression (3.3) is rewritten as (3.4).

$$g(n) = \frac{B(n)}{\| B(n) \|^2 + \lambda_1}\left(1 - \beta \frac{|e(n-1)|}{|\alpha(n)| + \lambda_2}\right) \tag{3.4}$$

where λ_1, λ_2 are small positive numbers. The error $e(n)$ will not converge to zero if the adaptive gain $g(n)$ is adjusted using expression (3.5) [9]. However, it will converge to a ball centred at the origin of the error space with radius of the ball relies on the values of λ_1, λ_2. Smaller values of λ_1, λ_2 contribute smaller error.

3.2 Reflection Coefficients Adaptation Algorithm

The updated algorithm for the reflection coefficients can be summarized as follow:

$$k_i(n) = k_i(n-1) - \eta_i(n) r_i(n) \tag{3.5}$$

$$\eta_i(n) = f_{i-1}(n) + k_i(n-1) b_{i-1}(n-1) \tag{3.6}$$

$$r_i(n) = \frac{b_{i-1}(n-1)}{\| b_{i-1}(n-1) \|^2}\left(1 - \tau \frac{|f_i(n-1)|}{|\eta_i(n)^2|}\right), \text{ where } 0 \le \sum_{i=1}^{p} \tau_i < 1 \tag{3.7}$$

The expression (3.7) is modified as (3.8) to prevent singularities.

$$r_i(n) = \frac{b_{i-1}(n-1)}{\| b_{i-1}(n-1) \|^2 + \mu_1}\left(1 - \tau \frac{|f_i(n-1)|}{|\eta_i(n)| + \mu_2}\right) \tag{3.8}$$

Again μ_1, μ_2 are small positive numbers.

4.THE DESIGN OF LATTICE-LADDER PREDICTOR USING LYAPUNOV STABILITY THEORY

The design of the lattice-ladder based on Lyapunov theory predictor is described by *Theorem 4.1* and *Theorem 4.2.*

4.1 The design of Ladder Weights Adaptation Algorithm

Theorem 4.1: For the given $B(n)$, if the weight vector $V(n)$ of the predictor $y^*(n) = V^T(n)B(n)$ is updated as follows

$$V(n) = V(n-1) + g(n)\alpha(n)$$

and

$$g(n) = \frac{B(n)}{\| B(n) \|^2}\left(1 - \beta \frac{|e(n-1)|}{|\alpha(n)|}\right) \tag{4.1}$$

where $0 \le \beta < 1$, then the prediction error $e(n)$ (2.5) asymptotically converges to zero.

Proof: Define a Lyapunov function of error $e(n)$

$$F(n) = e^2(n) \tag{4.2}$$

$$\begin{aligned}
\Delta F(n) &= F(n) - F(n-1) = e^2(n) - e^2(n-1) \\
&= (y(n) - V^T(n)B(n))^2 - e^2(n-1) \\
&= (y(n) - (V^T(n-1) + g^T(n)\alpha(n))B(n))^2 - e^2(n-1) \\
&= (y(n) - V^T(n-1)B(n) - g^T(n)\alpha(n)B(n))^2 - e^2(n-1) \\
&= (\alpha(n) - g(n)\alpha(n)B(n))^2 - e^2(n-1) \\
&= \alpha^2(n)\left(1 - g^T(n)B(n)\right)^2 - e^2(n-1)
\end{aligned} \tag{4.3}$$

Using expression (4.1) in expression (4.3), we have

$$\Delta F(n) = -(1-\beta^2)e^2(n-1) < 0 \tag{4.4}$$

4.2 The design of Reflection Coefficients Adaptation Algorithm

Theorem 4.2: For the given $f_i(n)$ and $b_{i-1}(n-1)$, if the reflection coefficients, $k_i(n)$ is updated as follows

$$k_i(n) = k_i(n-1) - \eta_i(n) r_i(n)$$

and $$r_i(n) = \frac{b_{i-1}(n-1)}{\| b_{i-1}(n-1) \|^2}\left(1 - \tau \frac{| f_i(n-1) |}{| \eta_i(n)^2 |}\right) \tag{4.5}$$

where $0 \le \sum_{i=1}^{P} \tau_i < 1$, then sum of forward prediction errors $\sum_{i=1}^{P} f_i(n)$ asymptotically converges to zero.

Proof: Define a Lyapunov function of sum of forward prediction errors $\sum_{i=1}^{P} f_i(n)$

$$G(n) = \sum_{i=1}^{P} f_i^2(n) = \left(\sum_{i=1}^{P} f_i(n)\right)^2 \tag{4.6}$$

$$\Delta G(n) = \left(\sum_{i=1}^{P} f_i(n)\right)^2 - \left(\sum_{i=1}^{P} f_i(n-1)\right)^2 = \left(\sum_{i=1}^{P} f_i(n-1) + k_i(n) b_{i-1}(n-1)\right)^2 - \left(\sum_{i=1}^{P} f_i(n-1)\right)^2$$

$$= \left(\sum_{i=1}^{P} f_i(n-1) + k_i(n-1) b_{i-1}(n-1) - \eta_i(n) r_i(n) b_{i-1}(n-1)\right)^2 - \left(\sum_{i=1}^{P} f_i(n-1)\right)^2$$

$$= \left(\sum_{i=1}^{P} \eta_i(n) - \eta_i(n) r_i(n) b_{i-1}(n-1)\right)^2 - \left(\sum_{i=1}^{P} f_i(n-1)\right)^2$$

$$= \left(\sum_{i=1}^{P} \eta_i(n)(1 - r_i(n) b_{i-1}(n-1))\right)^2 - \left(\sum_{i=1}^{P} f_i(n-1)\right)^2 \tag{4.7}$$

Using expression (4.5) in expression (4.7), we have

$$\Delta G(n) = -\left(1 - \left(\sum_{i=1}^{P} \tau\right)^2\right)\left(\sum_{i=1}^{P} f_i(n-1)\right)^2 < 0 \tag{4.8}$$

5. SIMULATION

In this section, we illustrate the application of the nonlinear adaptive prediction described herein to two speech signals, which are denoted *S1* and *S2*. Signal *S2* is identical to that used by Haykin and Li [8]. Those signals are available from the WWW homepage [12].

Figure. 3, 4 show the plots of 10000 samples of the speech signals *S1* and *S2* versus time. Figure. 5, 6 illustrate the plots of 10000 samples of the square predictor error or the difference between the estimated and current input values, $e^2(n)$ of the proposed lattice-ladder predictor for *S1, S2* respectively. Simulation results have shown the lattice-ladder predictor has fast error convergence property. In addition it is highly tolerate the noise introduced by the quantizer error and has better noise resistance performance.

6. CONCLUSION

A new adaptive lattice-ladder predictor for nonstationary speech signals is proposed. The lattice reflection coefficients and the ladder weights are adaptively adjusted by the algorithms that are designed using Lyapunov theory so that the error can converge asymptotically. The stochastic properties of the signals are not required and the stability of prediction error is guaranteed by the Lyapunov Stability Theory The proposed predictor has the potential for solving difficult task in the circumstance where nonlinearity and nonstationary are both important factor. The proposed scheme can be applied to the adaptive predictive coding techniques such as DPCM or ADPCM. The latter applications have yet to be explored. Simulation examples have demonstrated the excellent convergence property and robustness to nonstationary signals based on the new predictor design.

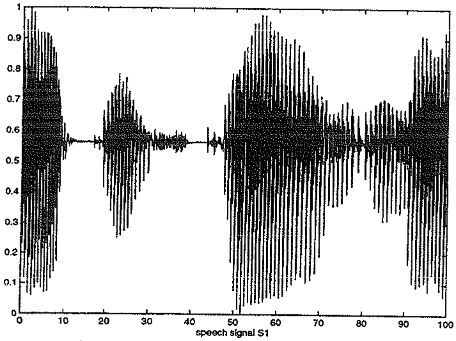

Figure 3: The speech signal S1

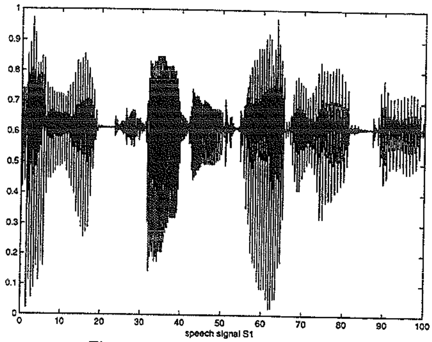

Figure 4: The speech signal S2

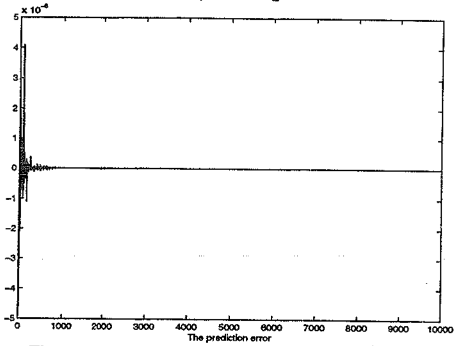

Figure 5: The prediction error for S1($x10^{-8}$)

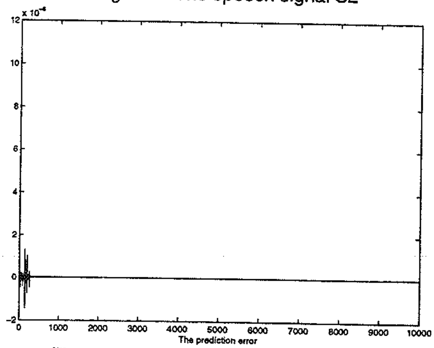

Figure 6: The prediction error for S2($x10^{-8}$)

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